8.6 – Introduction to Rational Functions.

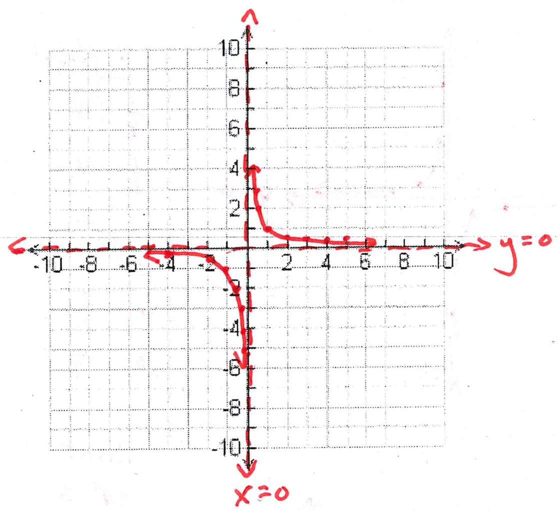
Objectives:

1. Investigate inverse variations.
2. Define rational functions.
3. Graph and find the equations of transformations of the parent function $y = \frac{1}{x}$.
4. Use rational functions to solve mixture problems and for other applications.

Rational Function

A **rational function** is one that can be written as a quotient, $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are both polynomial expressions. The denominator polynomial cannot equal the constant 0.

Graph the parent rational function: $f(x) = \frac{1}{x}$



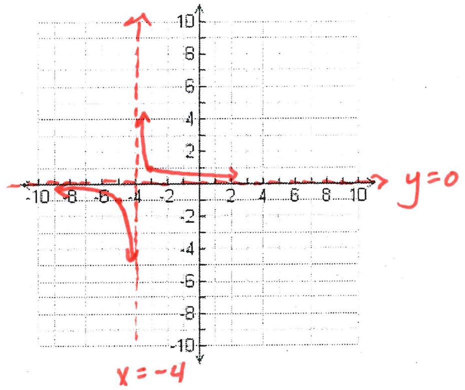
Asymptote: A line that a graph *approaches* but never crosses as the magnitude of the x- or y-values increases without bound.

x	$f(x)$
-4	$-\frac{1}{4}$
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$-\frac{1}{5}$	-5

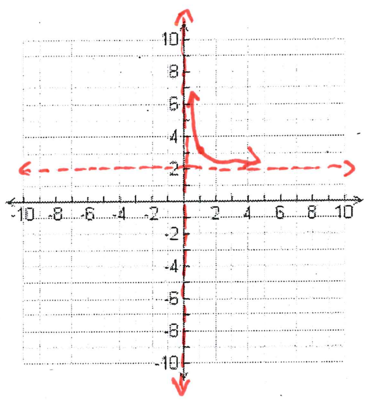
x	$f(x)$
$\frac{1}{5}$	5
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
4	$\frac{1}{4}$

Example 1: Describe the transformations of each rational function from the parent function $f(x)=\dfrac{1}{x}$. Then graph the new function. Be sure to identify any asymptotes:

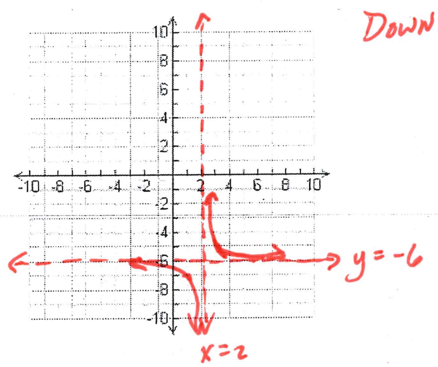
a. $h(x)=\dfrac{1}{x+4}$ LEFT 4



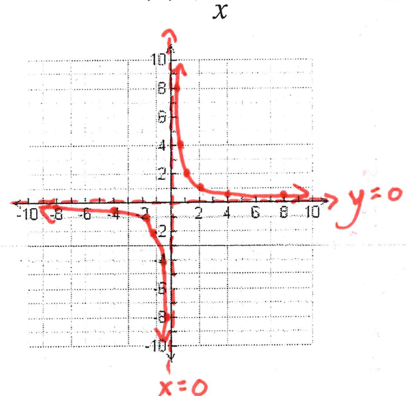
b. $h(x)=\dfrac{1}{x}+2$ UP 2



c. $h(x)=\dfrac{1}{x-2}-6$ RIGHT 2 DOWN 6

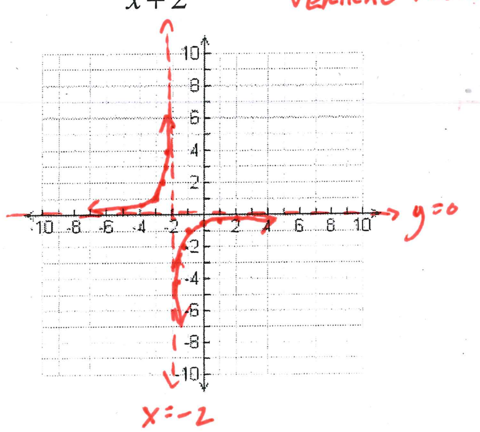


d. $h(x)=\dfrac{2}{x}$ VERTICAL STRETCH SF 2

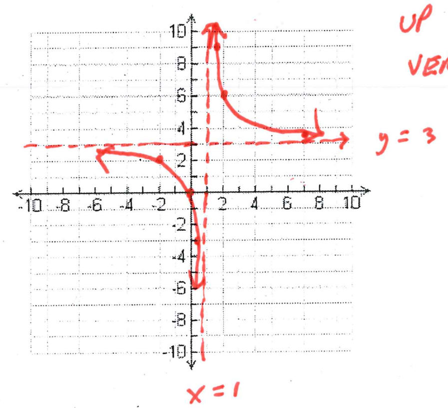


x	h(x)
$\frac{1}{4}$	8
$\frac{1}{2}$	4
1	2
2	1
4	$\frac{1}{2}$
8	$\frac{1}{4}$

e. $h(x)=\dfrac{-1}{x+2}$ LEFT 2 VERTICAL REFLECTION



f. $h(x)=\dfrac{3}{x-1}+3$ RIGHT 1 UP 3 VERT STRETCH SF 3



2	6
7	$3\frac{1}{2}$
$1\frac{1}{2}$	9
0	0
-2	2
$\frac{1}{2}$	-3

Write equations for the asymptotes of each hyperbola.

a. $y = \frac{2}{x}$

$x=0$

$y=0$

b. $y = \frac{1}{x+3}$

$x=-3$

$y=0$

c. $y = \frac{1}{x} + 5$

$x=0$

$y=5$

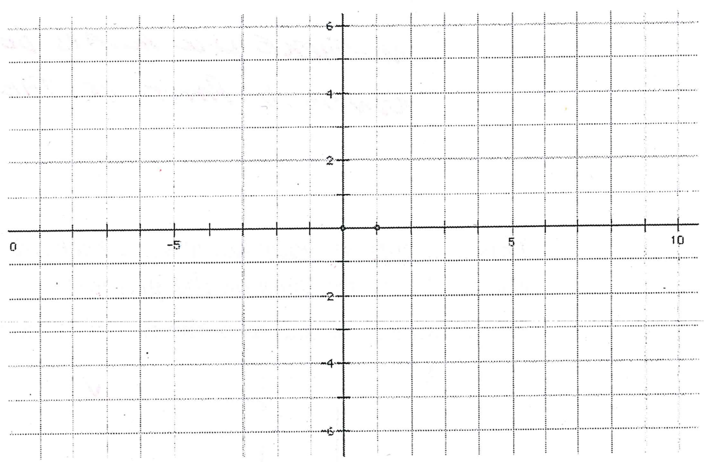
d. $y = \frac{1}{x-2} - 6$

$x=2$

$y=-6$

Example 3: Describe the function $f(x) = \frac{2x-5}{x-1}$ as a transformation of the parent function $f(x) = \frac{1}{x}$. Then sketch a graph. **[challenge]**

SKIP



3. Solve.

a. $\frac{6}{x-5} = -2$

$\frac{6}{(x-5)} = -\frac{2}{1}$

$-2(x-5) = 6 \cdot 1$

$-2x + 10 = 6$

$-10 \quad -10$

$\frac{-2x}{-2} = \frac{-4}{-2}$

$\boxed{x=2}$

30% of 100 = .3 × 100 = 30

70% of 100 = .7 × 100 = 70

Example B: Suppose you have 100 mL of a solution that is 30% acid and 70% water. How many mL of acid do you need to add to make a solution that is 60% acid?

ACID / TOTAL SOL $\frac{30+x}{100+x} = \frac{.6}{1}$

$.6(100+x) = 30+x$

$60 + .6x = 30 + x$

$-.6x \qquad -.6x$

$60 = 30 + .4x$

$-30 \quad -30$

$\frac{30}{.4} = \frac{.4x}{.4}$

$\boxed{75\text{ mL} = x}$

To make it 90% acid?

$\frac{30+x}{100+x} = \frac{.9}{1}$

$.9(100+x) = 1(30+x)$

$90 + .9x = 30 + x$

$-30 \qquad -30$

$60 + .9x = 1x$

$-.9x \quad -.9x$

$\frac{60}{.1} = \frac{.1x}{.1}$

$600 = x$

$\boxed{600\text{ mL}}$

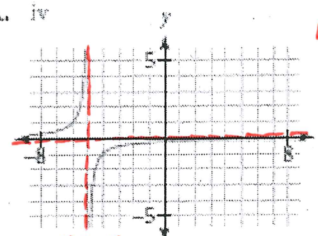
Can it ever be 100% acid?

NO - THERE WILL ALWAYS BE SOME OF THE NON-ACID PART OF THE SOLUTION

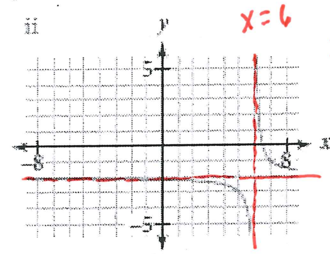
This is a copy of your homework exercise #7. It may be easier to determine the equation if you sketch the asymptotes on the graphs.

7. Match the graphs of the rational functions with their equations.

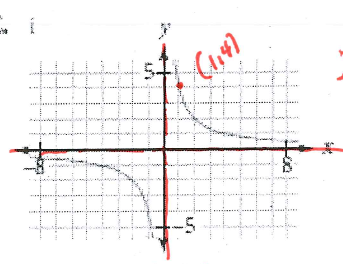
A. iv IV



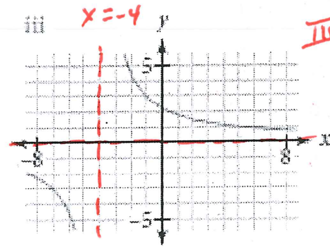
B. ii II



C. i I



D. iii III



i. $f(x) = \frac{4}{x}$ — VERTICAL STRETCH SF 4

ii. $f(x) = \frac{1}{x-6} - 2$ — RIGHT 6 DOWN 2

iii. $f(x) = \frac{9}{x+4}$ — LEFT 4 VERT STRETCH SF 9

iv. $f(x) = \frac{-1}{x+5}$ — LEFT 5 REFLECTED ACROSS X